This quiz key contains bonus points

(for identifying singular solutions and

completely simplifying final answers).

Do not use negative grading (assuming

you have 30 points and deducting points

for errors and omissions).

[2] $3x^2y\frac{dy}{dx} - 2x(x-2)y^2 = 3(2x+1)y^{\frac{1}{2}}$

IS $y=0$ A SOLN? YES
$3x^2(0)(0) - 2x(x-2)(0) = 3(2x+1)(0)$

(½) $\frac{dy}{dx} - \frac{2x(x-2)}{3x^2}y = \frac{2x+1}{x^2}y^{-\frac{1}{2}}$

BERNOULLI $n=-\frac{1}{2}$

$v = y^{1-\left(-\frac{1}{2}\right)} = y^{\frac{3}{2}}$ (½)

$\frac{dv}{dx} = \frac{3}{2}y^{\frac{1}{2}}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2}{3}y^{-\frac{1}{2}}\frac{dv}{dx}$

(½) $\frac{2}{3}y^{-\frac{1}{2}}\frac{dv}{dx} - \frac{2x(x-2)}{3x^2}y = \frac{2x+1}{x^2}y^{-\frac{1}{2}}$

$\frac{dv}{dx} - \frac{x(x-2)}{x^2}y^{\frac{3}{2}} = \frac{3(2x+1)}{2x^2}$

(½) $\frac{dv}{dx} - \frac{x(x-2)}{x^2}v = \frac{3(2x+1)}{2x^2}$ ← MANDATORY CHECKPOINT! (½) LINEAR

$u = e^{\int -\frac{x(x-2)}{x^2}dx} = e^{\int\left(-1+\frac{2}{x}\right)dx} = e^{-x+2\ln|x|} = x^2e^{-x}$ (½)

(½) $x^2e^{-x}\frac{dv}{dx} - x(x-2)e^{-x}v = \frac{3}{2}(2x+1)e^{-x}$ ← MANDATORY CHECKPOINT: (½)

$\frac{d}{dx}x^2e^{-x} = 2xe^{-x} + x^2(-e^{-x})$

$= xe^{-x}(2-x)$ ✓

$x^2e^{-x}v = \int\frac{3}{2}(2x+1)e^{-x}dx + C$

(½) $x^2e^{-x}v = -\frac{3}{2}(2x+3)e^{-x} + C$

$v = -\frac{3(2x+3)}{2x^2} + \frac{Ce^x}{x^2}$

u	dv	
$2x+1$	e^{-x}	+
2	$-e^{-x}$	−
0	e^{-x}	

(½) OR EQUIVALENT

$-(2x+1)e^{-x} - 2e^{-x}$

$= -(2x+3)e^{-x}$

(½) $y^{\frac{3}{2}} = \frac{Ce^x - 6x - 9}{2x^2}$

(½) $y = \left(\frac{Ce^x - 6x - 9}{2x^2}\right)^{\frac{2}{3}}$

(½) $y=0$ IS A SINGULAR SOLUTION (NOT PART OF THIS FAMILY)

[3] $(x^2-xy-y^2)\,dx + (x^2+2xy)\,dy = 0$ (½)

$y = vx \longrightarrow dy = v\,dx + x\,dv$

(½) $(x^2 - vx^2 - v^2x^2)\,dx + (x^2 + 2vx^2)(v\,dx + x\,dv) = 0$

$(tx)^2 - (tx)(ty) - (ty)^2 = t^2(x^2 - xy + y^2)$

$(tx)^2 + 2(tx)(ty) = t^2(x^2 + 2xy)$ (½)

BOTH HOMOGENEOUS ORDER 2

$(1 - v - v^2)\,dx + (1+2v)(v\,dx + x\,dv) = 0$

$(1 - \cancel{v} - v^2 + \cancel{v} + 2v^2)\,dx + x(1+2v)\,dv = 0$

(½) $(1+v^2)\,dx + x(1+2v)\,dv = 0$

(½) $\int \frac{1+2v}{1+v^2}\,dv = \int -\frac{1}{x}\,dx$ ← MANDATORY CHECKPOINT: SEPARABLE (½)

$\int \left(\frac{1}{1+v^2} + \frac{2v}{1+v^2}\right) dv = -\ln|x| + C$

$u = 1+v^2$ (½)

$\int \frac{1}{u}\,du = \ln|u|$

(1) $\tan^{-1} v + \ln(1+v^2) = -\ln|x| + C$ (½)

(½) $\tan^{-1}\frac{y}{x} + \ln\left(1 + \frac{y^2}{x^2}\right) = -\ln|x| + C$

$\tan^{-1}\frac{y}{x} + \ln\left(\frac{x^2+y^2}{x^2}\right) = -\ln|x| + C$

$\tan^{-1}\frac{y}{x} + \ln(x^2+y^2) - \ln|x|^2 + \ln|x| = C$

$\tan^{-1}\frac{y}{x} + \ln(x^2+y^2) - 2\ln|x| + \ln|x| = C$

(½) $\tan^{-1}\frac{y}{x} + \ln(x^2+y^2) - \ln|x| = C$

$\frac{e^{\tan^{-1}\frac{y}{x}} \cdot (x^2+y^2)}{|x|} = C$

TALK TO ME IF YOU USED $x = vy$ (MUCH HARDER)

(½) $(x^2+y^2)\,e^{\tan^{-1}\frac{y}{x}} = Cx$

[4] $\underbrace{(8x^{n+2}e^{ky} - 10x^{n+5}e^{ky} - 6x^{n+1}e^{(k+5)y})}_{P}dy + \underbrace{(15x^{n+4}e^{ky} + 2x^{n}e^{(k+5)y})}_{Q}dx = 0$ (1/2)

★ MEANS MUST HAVE BOTH PARTS CORRECT

$P_x = 8(n+2)x^{n+1}e^{ky} - 10(n+5)x^{n+4}e^{ky} - 6(n+1)x^{n}e^{(k+5)y}$ ★ (1/2)

$Q_y = 15k\,x^{n+4}e^{ky} + 2(k+5)x^{n}e^{(k+5)y}$ (1/2)

(1/2) $8(n+2) = 0$ $\quad -10(n+5) = 15k$ $\quad -6(n+1) = 2(k+5)$

$n = -2$ $\quad -30 = 15k$ $\quad 6 = 6$ ✓

$k = -2$

(1/2) $\mu = x^{-2}e^{-2y}$

(1/2) $\underbrace{(8e^{-2y} - 10x^{3}e^{-2y} - 6x^{-1}e^{3y})}_{M}dy + \underbrace{(15x^{2}e^{-2y} + 2x^{-2}e^{3y})}_{N}dx = 0$

★ (1/2) $M_x = -30x^{2}e^{-2y} + 6x^{-2}e^{3y}$

★ (1/2) $N_y = 15x^{2}e^{-2y}(-2) + 2x^{-2}e^{3y}(3) = -30x^{2}e^{-2y} + 6x^{-2}e^{3y} = M_x$

MANDATORY CHECKPOINT: EXACT (1/2)

$F = \int (15x^{2}e^{-2y} + 2x^{-2}e^{3y})\,dx$

$= 5x^{3}e^{-2y} - 2x^{-1}e^{3y} + C(y)$ (1/2)

$F_y = 5x^{3}e^{-2y}(-2) - 2x^{-1}e^{3y}(3) + C'(y)$ (1/2)

$= -10x^{3}e^{-2y} - 6x^{-1}e^{3y} + C'(y) = 8e^{-2y} - 10x^{3}e^{-2y} - 6x^{-1}e^{3y}$

★ (1/2) $C'(y) = 8e^{-2y}$

$C(y) = -4e^{-2y}$

MANDATORY CHECKPOINT: ONLY y (1/2)

$5x^{3}e^{-2y} - 2x^{-1}e^{3y} - 4e^{-2y} = C \rightarrow 5x^{4} - 2e^{5y} - 4x = Cxe^{2y}$

(1/2) NO POINTS IF MISSING "$= C$" (1/2)